Longitudinal profile of $N_\mu/N_e$ in extensive air showers: Implications for cosmic rays mass composition study

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Abstract
$N_\mu/N_e$, muon to electron population ratio in extensive air showers at high altitudes has been shown to be a suitable estimator of primary cosmic rays mass composition. This study is based on simulated extensive air showers. The $N_\mu/N_e$ ratio has been obtained in 100 depths from the top of the atmosphere to the sea level for different primary particle masses and energies. An empirical relation between cosmic ray atomic mass and $N_\mu/N_e$ has been obtained. The relation has then been used for estimation of atomic masses of progenitors of another set of simulated showers. Although the estimated masses are rough, the accuracy of the estimation improves with observation altitude.

Keywords: cosmic rays, extensive air showers, cosmic ray mass composition, muon to electron ratio

1. Introduction
Cosmic rays (CR) are charged particles, mainly protons and other nuclei, with kinetic energies higher than their rest energies. They are supposed to be produced in high energy astrophysical sites such as supernovae. The energy spectrum of CRs is proportional to $E^{-2.7}$, in which $E$ is the kinetic energy. The spectrum is so steep that we only receive one particle per square meter per year at $E = 10^{15}$ eV. It is not possible to observe such rare events and to determine CR mass and energy directly, using usual particle detectors. On the other hand, when these particles enter the atmosphere, they generate a huge number of secondary particles after interacting with air nuclei. This phenomenon is known as extensive air shower (EAS) [1]. The majority of these secondary particles near the surface of the earth are electrons and muons. In this paper, electrons and positrons are referred to as electrons. The interaction of a primary particle with a nucleus of an air atom should occur at high altitudes near the top of the atmosphere. The first sequence of interactions produces high energy hadrons (mainly pions), which are not stable. They eventually decay into muons. These high energy muons can feed the electron population in the shower by their subsequent interactions, e.g. bremsstrahlung and pair production. The shower front can cover a large area ($\sim 10^4$ to $10^6$ m$^2$). Because of the low loss rate, the population of muons in the shower grows up to a maximum and remains high even at sea level. In figure 1, longitudinal profiles of muon population, $N_\mu$, for some simulated showers have been shown. On the other hand, electrons are generated via pair production and bremsstrahlung of higher energy charge particles. Since the loss rate is higher for electrons, their population decreases considerably after a maximum, somewhere in the middle of the way down to sea level (see figure 2). One should then expect the variation of the $N_\mu/N_e$ ratio along the shower path.

Although one can derive analytical equations for the development of an EAS, the results of such equations only describe the average behavior of the shower. Many random processes occur in the development of an EAS so that there are considerable fluctuations in shower properties. The inherent randomness of EASs somehow sterilizes the analytical approach. Therefore, theoretical studies on EAS are often based on Monte Carlo simulations. Experimental studies on EASs are limited to detecting samples of secondary particles in the showers at the earth surface. The measurements, including particle density at detectors, and arrival times, should be compared to the results from simulations, in order to reconstruct the EAS, and estimate energy, mass, and direction of the primary CR. Since the size of a typical EAS spans a few kilometers in height, and hundreds of meters in width, it is practically impossible to do
experimental studies on the whole structure of EASs. Another aspect of EASs is their high energy. Terrestrial particle accelerators have maximum energy records around $E \sim 1$ TeV, which is far below the energy of the highest energy CRs which generate the EASs. Despite the lack of experimental verification, people have to rely on theoretical models tailored to high energy particle interaction cross sections. For the limitations mentioned above, most of research in high energy CR and EAS is based on Monte Carlo simulations [2-5].

In previous studies, the longitudinal profile of $N_e$ or $N_\mu$ has been investigated by simulation techniques [6, 7]. The dependence of the profiles on the mass composition of primary cosmic rays has been confirmed by other works [8]. It has been shown that $N_\mu/N_e$ of EAS can be used for estimation of cosmic ray mass composition [9]. Experimental studies are possible in some EAS arrays, which can estimate the electron size ($N_e$) and muon size ($N_\mu$) of individual showers. These kind of arrays have muon detectors which can filter out electrons. Such an array samples particle densities in the shower front. The data then is fitted into a theoretical model of shower front lateral profile. Shower size and core position are estimated from the fit. In the present work, we will see that higher altitude EAS arrays are more appropriate to study $N_\mu/N_e$ for cosmic ray mass composition determination.

2. Simulation of EAS

Among the computer programs developed for EAS simulation, CORSIKA [10] is the most applied code in the cosmic ray studies. In the present work, version 6.7.2 of the code has been used. The code has several options, including high and low energy model for hadronic interactions. For the large part of this work, default options, QGSJET01 model for high energy, and GHEISHA model for low energy hadronic interactions have been used. In order to check the validity of results with other hadronic interaction models, we also used NEXUS model for high, and URQMD model for low energy hadronic interactions in some EASs. The code can simulate electromagnetic processes carefully if EGS4 model is chosen. In order to reduce computation time, one can disable this option and use NKG analytical model for electromagnetic processes, but the results will not show inherent fluctuations of the EAS and only give average values. Since we have to take these fluctuations into account, EGS4 were enabled during simulations. Simulation time is proportional to the number of secondary particles generated in EAS. The number is proportional to shower energy, i.e. the energy of primary particle. On a 2.8 GHz CPU, the simulation of a shower initiated by a $10^{15}$ eV proton takes about 16 minutes. A $10^{18}$ eV shower would take 11 days on the same PC. Fortunately, there is a thinning option which can reduce the simulation time considerably. In this work, thinning has been used for $10^{16-10^{18}}$ eV showers. Primary cosmic rays selected for simulations include photon, proton, and nuclei of Helium, Oxygen, Silicon, and Iron, with energies from $10^{12}$ to $10^{18}$ eV. For each primary type and energy, 200 showers have been generated. Half of the showers were used to obtain an empirical relation between cosmic ray atomic mass $A$ and $N_\mu/N_e$. The other half of the data were used to test the model obtained in the previous step. CORSIKA can produce shower longitudinal profile output, which contains the number of various particles at predefined steps of atmospheric depths. In our data, step size was 10 g/cm$^2$ so that the number of different secondaries at 105 depths from the top of the atmosphere could be produced.

3. Results and discussion

From these data one can derive $N_\mu/N_e$ ratio at each atmospheric depth. Since we had 100 showers for each primary cosmic ray type and energy, an average and standard deviation of $N_\mu/N_e$ ratio have been obtained for each depth. As an example, the variation of $N_\mu/N_e$ ratio with atmospheric depth and primary particle type for $10^{15}$ eV showers have been presented in figure 3. In our results, energy threshold for electrons was 3 MeV,
that of muons was 1 GeV. These thresholds are typical in EAS array experiments [11, 12]. For a fixed shower energy, some ranges of depths in the atmosphere, at which different primaries can be distinguished by their $N_{\mu}/N_e$ ratio, could be found. In other words, one can compare the precision of $N_e/N_{\mu}$ based cosmic ray mass estimation at different observation levels. To try the idea, we considered using atmospheric depths, corresponding to KASCADE [11], Tibet array [12], Alborz Experiment [13], and the array at Sharif University of Technology [14]. They are almost at 1050, 600, 750, and 890 g/cm$^2$, respectively. The $N_{\mu}/N_e$ ratios for these depths are shown for some showers in figure 4. According to these plots, the highest altitude provides the best separation between different primary cosmic rays. It is worth noting that for the EASs considered here, the number of electrons in a shower at depth of Tibet is several times higher than those at KASCADE, while the difference in mean numbers is not considerable (see figures 1 and 2). Therefore, the muon to electron ratio is lower in Tibet (figure 4a). Because of higher values for $N_e$, relative error in $N_e$ measurement should be lower in Tibet. Since the absolute error in $N_{\mu}/N_e$ is proportional to $\sqrt{(\Delta N_{\mu}/N_{\mu})^2 + (\Delta N_e/N_e)^2}$, one would expect more accurate measurements for $N_{\mu}/N_e$ in Tibet. In figure 5, the variations of $N_e/N_{\mu}$ with primary particle mass for these sites are estimated by previous estimator function. For a fix shower energy and observation depth, the data can be fitted into a power law relation:

$$A = a \left( \frac{N_{\mu}}{N_e} \right)^b,$$

where $A$ is atomic mass of the primary particle, and $a$ and $b$ are fit parameters. The parameter values have been shown in table 1. This empirical relation can be applied for the estimation of the atomic mass of the primary cosmic ray of individual showers. Another set of simulated showers has been examined using this estimator. All gamma ray initiated showers are well separated from other cosmic ray showers by this technique. For most of these gamma ray showers, $A = 0$ has been obtained, and few showers have $0 < A < 0.1$. In figure 6, histograms of estimated primary masses for showers initiated by proton, Oxygen, and Iron nuclei are compared. Despite the wide distribution of estimated masses, the technique can successfully discriminate protons from particles heavier than Oxygen. As one can see, the higher the observation level is, the closer the estimated masses are to the actual ones. Even at the highest altitude, this technique ceases to give correct results for low energy ($E < 10^{13}$ eV for Tibet), as well as high energy ($E > 10^{16}$ eV for Tibet) showers. Figure 7 shows the results for different energies and altitudes. The data also show that the reliable energy range decreases at lower altitudes.

It has been shown that the application of different hadronic interaction models in EAS simulations has a minor effect on $N_e$, while it causes about 10% difference in $N_{\mu}$ [15]. In order to check the validity of our mass estimator, we have simulated a set of EASs, using NEXUS model for high energy and URQMD model for low energy hadronic interactions. Then, the masses of primaries were estimated by previous estimator function. Results have been presented in figure 8. It is worth noting that the parameters ($a$ and $b$) used here for showers generated by NEXUS-URQMD model were those obtained from showers generated by QGSJET01-GHEISHA models. Despite the change in interaction models, one can see that the estimator almost works the same as before. This shows that the validity of the technique does not critically depend on hadronic interaction models. More importantly, one can still conclude that higher altitude EAS arrays provide more accurate mass estimations.

In this work, we used the $N_e$ and $N_{\mu}$, which were provided by EAS generator code without considering the uncertainties caused by shower detection techniques. In EAS detector arrays, $N_e$ and $N_{\mu}$ are estimated by sampling the shower front at each detector. One should take the errors of such estimations into consideration. It has been claimed that for showers with $N_{e} > 10^6$, experiments like KASCADE, can estimate $N_e$ and $N_{\mu}$ with less than 20% error [16]. Almost the same results have been reported for simulations with a 16 detector array [17]. From equation 1, one can derive the error for estimated mass:

$$\frac{\Delta A}{A} = b \left( \sqrt{\left(\frac{\Delta N_e}{N_e}\right)^2 + \left(\frac{\Delta N_{\mu}}{N_{\mu}}\right)^2} \right).$$

For $\frac{\Delta N_e}{N_e} \approx \frac{\Delta N_{\mu}}{N_{\mu}} \approx 0.2$, one obtains $\frac{\Delta A}{A} \approx 0.3b$.

Minimum values of $b$ ($= 2$) give a 60% error in $A$. Errors of this size are less than statistical deviations in our
Figure 4. (color online) Longitudinal profile of muon to electron population ratio for $10^{15}$ eV showers at different depths in the atmosphere. Figure (a) to (d) correspond to altitudes near Tibet, Alborz, Sharif University, and KASCADE arrays, respectively. Each data point is an average of $N_\mu/N_e$ for 100 showers, and error bars are standard deviations.

Figure 5. Variation of $N_\mu/N_e$ with primary cosmic rays mass for $10^{15}$ eV showers at elevations corresponding to Tibet, Alborz, Tehran, and KASCADE arrays.
work. Therefore, mass estimation with experimental $N_\mu/N_e$ is still possible within tolerable errors.

4. Conclusion
Because of their low fluxes, high energy cosmic rays can only be detected trough their EAS. Most of EAS experiments can estimate $N_e$ and $N_\mu$ of these showers. The randomness inherent to EASs produces considerable fluctuations, especially in the electron size of showers ($N_e$). This is the main cause of roughness of estimations in EAS studies, including the one presented in this paper. Despite the fluctuations, this work shows that observations in higher altitudes can reduce the fluctuations in estimation of cosmic rays mass composition. Cosmic rays mass estimation based on $N_\mu/N_e$ has been used in previous researches by other people. The present work is the first one which pays attention to the impact of observation level on the accuracy of the mass estimation. High altitude EAS experiments, like Tibet array, were known to have the advantage to detect lower energy showers. This work shows that they can also give better estimations for cosmic rays mass composition. The results presented in this paper can also motivate extra efforts for the establishment of Alborz observatory.
Figure 6. Histograms of estimated atomic mass of $10^{15}$ eV primary cosmic rays at atmospheric depths near Tibet, Alborz, Tehran, and KASCADE experiments. Each histogram is obtained by application of the empirical model (see text) to 100 showers. Primary particle type and observation site are shown on top of each histogram.

Figure 7. (color online) Results of mass estimation for different energies and altitudes. Data points are average values, and error bars are standard deviations of the results.
Figure 8. The same as figure 6 for EASs generated with NEXUS model in high energy, and URQMD model in low energy hadronic interactions.

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